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BOOK REVIEWS

Hydrodynamic and Magnetohydrodynamic Turbulent Flows: Modelling and Statistical Theory. By A. Yoshizawa. Kluwer, 1998, 410 pp. ISBN 07923 52254. £139.50

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The practical analysis of turbulent flows almost invariably starts with an averaging procedure, whereby the velocity field is decomposed into the sum of its (ensemble) mean part $\boldsymbol{U}(\boldsymbol{x},t)$ and its fluctuating part $\boldsymbol{u}(\boldsymbol{x},t)$. The equation for the evolution of $\boldsymbol{U}(\boldsymbol{x},t)$ is then just the Navier–Stokes equation, augmented by a term $\partial R_{ij}(\boldsymbol{x},t)/\partial x_j$ on the right-hand side; $R_{ij}(\boldsymbol{x},t) = -\langle u_i u_j \rangle$ is the Reynolds stress tensor, responsible, in conjunction with viscous diffusion, for the transport of mean momentum within the flow field, and to or from its boundaries. The central (closure) problem of turbulence is to find a plausible, if not rigorous, means of representing $R_{ij}(\boldsymbol{x},t)$ in terms of $\boldsymbol{U}(\boldsymbol{x},t)$ (perhaps through a sequence of ever-improving approximations) so that the mean flow equation may be rendered (at least in principle) soluble. One can of course construct, on the basis of the Navier–Stokes equations, an exact evolution equation for $R_{ij}(\boldsymbol{x},t)$; but this involves the third-order moment $\langle u_i u_j u_k \rangle$, and the closure problem persists in more complex form, at higher order.

The term 'modelling of turbulence' is used to describe the construction of any closure scheme that provides a deterministic set of evolution equations for mean quantities – deterministic in the double sense that random fluctuations have been averaged out, and that the number of 'unknowns' is equal to the order of the system of evolution equations constructed for their determination. These equations must of course be coupled with boundary conditions appropriate to the particular flow geometry under consideration; but the hope must be that a turbulence model, once constructed, will apply to a reasonably wide range of geometries and of parameters (e.g. Reynolds number) characterizing the flow, since otherwise one can have no confidence that the model will be applicable outside the range of circumstances for which laboratory validation is available. Even the simplest models involve a number of dimensionless constants, which are naturally chosen to provide a 'best fit' with experimental observation. Proponents of particular models tend to emphasize the goodness of fit to some (necessarily finite) range of experiments, but may be less prepared to consider the limitations of their models, or to admit the possibility that they may give wildly incorrect predictions if applied to flow situations far outside the range for which experimental validation is available.

The book under review seeks "to elucidate the mathematical structures of the current turbulence modelling and give a firmer statistical theoretical basis to it". No attempt is made to provide a comparative evaluation of different models in terms of their success (or lack of it) in describing flows for which experimental results are available. The aim is rather to examine the modelling process from a theoretical standpoint, and to attempt to establish some kind of bridge between, on the one hand, the more fundamental theories of turbulence and, on the other, the more urgent imperatives of engineering practice. This bridge, as constructed by Yoshizawa, is one that connects two swamps, and that is crossed at some peril!

There are ten chapters, of which the first two are introductory. Chapter 3 deals with small-scale aspects of turbulent flow, and includes a substantial section of 30 pages on Kraichnan's direct-interaction approximation (DIA), rather forbidding at this early stage of the book. Kraichnan's original paper on this subject (*J. Fluid Mech.*, vol. 5, 1959, p. 497) which Yoshizawa follows, must surely be one of the most impenetrable ever published in *JFM*! And yet, as Kraichnan himself recognized in the years that followed, the theory is flawed insofar as it fails to take account of the 'sweeping' of small eddies by large eddies. The even more impenetrable 'Lagrangian-history direct interaction approximation' LHDIA) (R. H. Kraichnan, *Phys. Fluids*, vol. 8, 1965, p. 575) sought to incorporate the sweeping effect. Yoshizawa makes passing reference to this theory; it apparently lurks behind some of the closure schemes described in subsequent chapters.

Chapter 4 on "Conventional turbulence modelling" starts by seeking to explain the statistical basis for the widely used $K\epsilon$-model of turbulence in which the mean flow equation is coupled with equations for the kinetic energy density K and the local rate of dissipation of turbulent energy ϵ. The ϵ-equation is presented with little justification (no criticism of Yoshizawa here – it is impossible to justify!), and one is left with a dreadful sense of unease concerning its excessively heuristic character. As pointed out by Yoshizawa, even for channel turbulence, any such 'turbulent viscosity' models fails to capture the simple fact that the three contributions to turbulent energy, $\langle u^2\rangle$, $\langle v^2\rangle$ and $\langle w^2\rangle$, are all unequal. Something better is needed. But the higher-order models (akin to models for non-Newtonian behaviour in laminar flow) become rapidly more complex in structure, while failing to yield anything in the way of physical understanding. Of course, the number of dimensionless constants begins to proliferate, so that a wide range of observed behaviours (e.g. the appearance of cross-stream flow components in pressure-driven turbulent flow along a duct of square cross-section) fall within the scope of such higher-order models; but the flexibility thus gained is coupled (for this reader) with a proportionate loss of plausibility.

Chapter 5 on "Subgrid-scale modelling" is an excellent introduction to this complex topic, but chapter 6 on "Two-scale direct interaction approximation" is extremely heavy going, and I can't claim to have gained much insight from it – no doubt my fault rather than that of the author, who battles on through an extraordinary swathe of equations, expansions, arbitrary closure assumptions and empirical estimation of constants, with great energy and tenacity, the only problem being that the ultimate destination remains shrouded in obscurity.

One point in this chapter troubled me particularly, in section (6.7.2) (on frame-rotation effects), where Yoshizawa states that "frame rotation brings a preferred direction along the axis of rotation, and the mirror symmetry of turbulence properties is lost even in isotropic turbulence". But surely the breaking of mirror symmetry requires more than just the Coriolis effects associated with frame rotation. It requires in addition the flux of some quantity (e.g. energy) with a non-zero component parallel to the axis of rotation (H. K. Moffatt, *J. Fluid Mech.*, vol. 44, 1970, p. 705). Yoshizawa uses the statement quoted above as motivation for an extended consideration of effects associated with helicity in turbulent flows – a heavy super-structure on a dubious foundation.

I jump to chapter 9, "Magnetohydrodynamic turbulence modelling", for which of course non-zero helicity is of central importance for the dynamo effect (i.e. the spontaneous generation of large-scale magnetic field). Yoshizawa arrives at the well-known alpha-effect by what to my mind are excessively convoluted arguments. At the same time, and more controversially, he identifies a novel effect associated with non-zero cross-helicity $\langle \boldsymbol{u} \cdot \boldsymbol{b} \rangle$, where $\boldsymbol{b}$ is the fluctuating ingredient of the magnetic field. This novel effect is that in a medium rotating with mean angular velocity $\boldsymbol{\Omega}$, a contribution to the mean electromotive force $\langle \boldsymbol{u} \wedge \boldsymbol{b} \rangle$ is generated parallel to $\boldsymbol{\Omega}$, with a coefficient that is a weighted integral of the cross-helicity spectrum (just as the α of the alpha-effect is, in the first-order smoothing approximation, a weighted integral of the helicity spectrum). In a later section (10.3) entitled "Cross-helicity dynamo", Yoshizawa shows how this effect, on its own, can generate a toroidal magnetic field in a rotating medium (e.g. a star); of course, the cross-helicity has to be there in the first place, and in order to have a genuine dynamo, the origin of this cross-helicity would also have to be established, a point that is not addressed in this treatment.

Yoshizawa's cross-helicity effect is quite puzzling for two reasons: first, it occurs in the absence of any mean magnetic field; second, as Yoshizawa recognizes, it is not invariant under change to a rotating frame of reference. How can a mean electromotive force be non-invariant under this change of frame, when the mean magnetic field is zero? The answer is perhaps concealed in the 'quasi-kinematic' approach through which the effect is revealed; in this kinematic theory, the statistics of the velocity field are regarded as given (and the manner in which these statistics change under change to a rotating frame of reference is not considered). If we think of a state of maximal cross-helicity in which, at time $t = 0$, $\boldsymbol{b}$ is everywhere parallel to $\boldsymbol{u}$, then it is easy to see that, under the induction equation of magnetohydrodynamics, $\boldsymbol{b}$ will tend to be rotated around the direction of $\boldsymbol{\Omega}$, whereas (within a kinematic framework) any corresponding effect on $\boldsymbol{u}$ is not taken into account. Thus, in this scenario, $\boldsymbol{b}$ does not remain everywhere parallel to $\boldsymbol{u}$, and a field $\langle \boldsymbol{u} \wedge \boldsymbol{b} \rangle$ parallel to $\boldsymbol{\Omega}$ is indeed systematically generated. Note however that there is a 'preferred' frame of reference (namely that in which the statistics of $\boldsymbol{u}$ are prescribed), and so the apparent dependence of the effect on the frame of reference becomes comprehensible. But the kinematic approach (whether 'quasi' or not) is artificial in a rotating medium, and one is left with the impression that the cross-helicity effect is a by-product of this artificiality. Nevertheless, it is thought-provoking, and a more detailed analysis might be illuminating.

All in all, I find this book a rather strange mixture, intensely theoretical and abstract at one extreme, and yet attempting to deal in a systematic way with the infinitely variable and irksomely unjustifiable procedures of modelling at the other. And then these highly debatable aspects of magnetohydrodynamic turbulence are thrown in for good measure. At best, it is a brave attempt to cover a horribly difficult field, of interest perhaps to experienced researchers in the field, but I would hesitate to recommend it for graduate students new to the subject of turbulence.

H. K. Moffatt

The Handbook of Fluid Dynamics. By R. W. Johnson. CRC Press, 1998. 1400 pp. ISBN 0849 325099. £99.00.

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Fundamentals of Fluid Mechanics. By J. A. Schetz & A. E. Fuhs. Wiley Interscience, 1999. 935 pp. ISBN 0471 348562. £111.00.

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Both these books are dangerous and threaten the health of Fluid Mechanics. They are both too ambitious in their aim of an overview of the subject and, by failing, they give the appearance of a subject that is failing. Any future compilation of this nature should have firm editorial leadership and be web-based.

When I was sent these books to review for *JFM*, my heart sank. *The Handbook* weighs 3.2 kg (7 lbs) and so is a misnomer to start with. *Fundamentals* is lighter but still has 960 pages of text. Faced with nearly 2500 pages of text to review, I did what I should not have done: nothing. Every now and then I would dip into one of the chapters (48 in *The Handbook*, 14 in *Fundamentals*) and came away frustrated. Whilst individual chapters are sometimes extremely good, I could not see how either text hung together. So I kept reading and became more frustrated. It was only recently that the penny dropped. Neither book works. Neither book should have seen the light of day.

The Handbook has one Editor, 6 Advisory Board members, 68 contributors and 42 reviewers for its 48 Chapters. Its scope could hardly be broader. Divided into 6 unequal parts (Basics, Classical Fluid Dynamics, High Reynolds Number Asymptotic Theories, Numerical Solution of the Equations of Fluid Dynamics, Experimental Methods in Fluid Mechanics, Applications), it also contains three appendices (Mathematics of Fluid Mechanics, Tables of Dimensionless Numbers, Properties of Gases and Vapours). The second part (Classical Fluid Dynamics) dominates the book (it has 17 Chapters all to itself) with Applications coming in a distant second with 10 Chapters. Individual Chapter titles vary considerably from ‘Mesoscale Oceanic Flows’ to ‘Laser-Doppler Velocimetry’ and from ‘Computer Science’ to ‘Incompressible Triple-Deck Theory’. There is also a Chapter on the history of the subject.

So what could be wrong with a text which, according to the Preface ‘. . . is to provide the entering . . . professional with a useful guide and reference to a broad range of areas in the field of Fluid Dynamics’?

A clue lies in the asterisk in the Global Nomenclature section that precedes the Contents. The reader is informed that ‘Individual authors may define their own nomenclatures which supersede the above’. Other clues lie in the Appendices. Appendix A (Mathematics of Fluid Mechanics) is 93 pages long. Yet by removing just 6 lines (dedicated to Reynolds Transport Theorem), the whole thing could easily be called ‘Mathematics of Solid Mechanics’ or even, at a pinch, ‘Mathematics of General Relativity’. This appendix is just a collection of mathematics, with nothing holding it together; just like the book. Appendix B (Tables of Dimensionless Numbers) looks better and a few numbers here were new to me. But where is the editorial effort linking the numbers to pages, sections or even Chapters in the book? All we get is just over 2 pages listing ‘Phenomena in which nondimensional parameters are applicable’. Note the minor editorial slip here: dimensionless numbers have suddenly become nondimensional parameters. Even this section is not original to the book but appeared before in 1995 (reprinted with permission).

Fundamentals is less broad in scope as its title implies. It focuses on what is claimed to be a need for ‘. . . a more thorough understanding of classic fluid theory and laws . . . ’ given ‘. . . the growing capabilities of computational fluid dynamics and the development of laser velocimeters and other new instrumentation . . . ’. Here we have 2 editors, 7 Editorial Review Board members and 39 contributors for 14 chapters. [Some of The Handbook contributors reappear here.] But what we have is not a new book at all but a distillation of the earlier *Handbook of Fluid Dynamics and Fluid Machinery* by the same editors, published in March 1996, currently still available at £556.

Apparently it had been decided that a single-volume revision was necessary and that this revision should emphasise ‘the most fundamental aspects of the subject’ (no doubt the drop in price to £111 may also have helped to boost sales). So *Fundamentals* is actually a subset of another book and the joins can be seen everywhere (e.g. in the Contents list, where Section I and its 14 Chapters is not followed by any Section II, and in the Preface where the Editors proudly state that the result (of the decision to slim down their original handbook) is ‘the present volume on *Fundamentals of Fluid Dynamics*’ [their italics]. It is not. It is ‘Fundamentals of Fluid *Mechanics*’ [my italics].

Why is all this dreadful and damaging to Fluid Mechanics? These volumes set themselves up to review the subject and they attract significant names to write for them. But the editorial side is lacking. Where is the vision of explaining exactly what does bind Fluid Mechanics together, of what precisely is needed mathematically, numerically and experimentally and why?

Whatever the future holds for such compilations, strong focused editorial leadership is absolutely necessary. Whether one or more Editor is crucial presumably depends on who is involved. But why in book form?

The Handbook alone weighs more than most laptops. Once that threshold has been passed, surely it is time to put things on the web? Several sites currently exist which contain educational material for students. Why not go the way of *Encyclopaedia Britannica* and create a comprehensive ‘Website of Fluid Mechanics’. As well as articles, Java applets could be used to show the power of numerics (from the hodograph transformation to full CFD simuations) and videos could also be included, as could snapshots of flows taken from Van Dyke’s *Album of Fluid Motion*. The community would provide the iterations needed to make the site really useful. Editors and webmasters for the site should come from amongst the *JFM* readership. Any volunteers?

S. J. HOGAN